THE MEASUREMENT PROCEDURE IN THE DILATOMETER METHOD OF DETERMINING THE THERMAL DIFFUSIVITY OF MATERIALS

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The optimum procedure for measuring the thermal diffusivity of materials using dilatometer methods is derived analytically. The theoretical conclusions obtained are illustrated with experimental data.

A serious disadvantage of dilatometer methods of determining the thermal diffusivity of materials is the correct choice of the kinetic stage of the measurements, since an exact correspondence between the fundamental theoretical relations and the actual thermal expansion process of the prismatic specimens being tested is only observed over a certain time interval [1], the beginning of which corresponds to the fact that the temperature of one of the bases of the prism is equal to the temperature of the heat-transfer medium (as a result of which there is a temperature jump at this end of the specimen [2]). The time interval t_1, during which measurements should not be made (i.e., the time which elapses from the instant when the end comes in contact with the medium until the end acquires its temperature) depends both on the heat-transfer coefficient α between the end and the medium, and on the heat-transfer parameters of the material of the prism, namely, the thermal conductivity λ and the thermal diffusivity a as given by the inequality [1]

$$a\left(\frac{\alpha}{\lambda}\right)^2 t_1 \gg 1 \tag{1}$$

with a systematic error less than $\lambda^2/2at_1\alpha^2$. In particular, by assigning the systematic error as 0.5%, we have, in the general case the following relation for the critical time interval t_1:

$$t_1 \geqslant \frac{100}{a}\,\frac{\lambda^2}{\alpha^2}\,. \tag{2}$$

On the other hand, the upper limit of the permissible kinetic measurement time t_2 must reflect the fact that thermal excitations propagate in a prism of finite length l according to the laws which hold for semiinfinite media, or, in other words, the measurement time must not exceed the time t_2 when the thermal excitations reach the end opposite to the heated or cooled end. The critical value of t_2 is given in [2], and is defined by the condition

$$\exp\left(-\frac{l^2}{4at_2}\right) \ll 1. \tag{3}$$

Thus, starting from the requirement that the systematic error must not exceed 0.1%, it is easy to obtain the following relation for t_2:

$$t_2 \leqslant \frac{l^2}{32a}\,. \tag{4}$$

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It follows from conditions (2) and (4) that the kinetic measurement interval Δt in the dilatometer method is limited by the condition

$$\frac{100}{a}\,\frac{\lambda^2}{\alpha^2} \leqslant \Delta t \leqslant \frac{l^2}{32a}\,. \tag{5}$$

Moreover, it follows from the critical inequalities (1) and (3) that these criteria cannot by any means always be satisfied simultaneously, but only in those cases when the specimen being investigated has a length which satisfies the requirement

$$\frac{\alpha l}{\lambda} \gg 1. \tag{6}$$

It is noteworthy that the main assumption when deriving criterion (6) is the fact that the instants t_1 and t_2 coincide. Physically this means that the measurement interval reduces to a point, i.e., the temperatures of the heat-transmitting medium and the heated end are equal at the instant when the temperature perturbations reach the opposite end. Within the limits of the errors mentioned above, a suitable situation occurs, as can easily be calculated, when $\alpha l/\lambda \geq 50$, or, changing to the Biot criterion

$$\mathrm{Bi} \gg 50. \tag{7}$$

The above discussion only hides an important drawback of the dilatometer method, namely, the need to operate with an overall heating (or cooling) time which is measured from the instant when contact is made between the end of the specimen and the heat-transmitting liquid, including the stage that precedes the measurement interval

$$t_1 \leqslant \Delta t \leqslant t_2. \tag{8}$$

Hence, the use of the theoretical relation derived previously [2]

$$\Delta l\,(t) = 2\beta\Delta T\sqrt{\frac{at}{\pi}} \tag{9}$$

(where β is the coefficient of thermal expansion of the prism; ΔT is the temperature jump at the heated (cooled) end; and $\Delta l(t)$ is the absolute thermal change in the dimensions of the specimen in the time t), is incorrect and involves large systematic errors in view of the fact that Eq. (9) only holds during the time interval (8). Hence, a change from an overall heating time t in the theoretical relations to time intervals limited by relation (8) should lead to a considerable increase in the accuracy with which the coefficient a can be determined.

When there is such an increase in accuracy it is of interest to compare the time intervals τ_1 and τ_2 that follow one after the other, at which the length of the specimen has the same value L, provided that both intervals τ_1 and τ_2 lie in the range given by (8). By calculating the length L from a certain value t to t + τ_2 we obtain

$$L = 2\beta\Delta T\sqrt{\frac{a}{\pi}}\left(\sqrt{t+\tau_2} - \sqrt{t}\,\right). \tag{10}$$

The length with the same value L after an interval τ_1 immediately preceding the interval τ_2 (i.e., the time from the interval $t-\tau_1$ to the instant t) can also be calculated from Eq. (9) and is given by

$$L = 2\beta\Delta T\sqrt{\frac{a}{\pi}}\left(\sqrt{t} - \sqrt{t-\tau_1}\right). \tag{11}$$

Substracting Eq. (11) from Eq. (10) and simplifying the result we obtain

$$2\sqrt{t} = \sqrt{t+\tau_2} + \sqrt{t-\tau_1}. \tag{12}$$

On the other hand, by adding Eqs. (10) and (11) we obtain

$$L = \beta\Delta T\sqrt{\frac{a}{\pi}}\left(\sqrt{t+\tau_2} - \sqrt{t-\tau_1}\right). \tag{13}$$

To proceed further we must square Eq. (13), thereby obtaining

$$L^2 = (\beta\Delta T)^2 \frac{a}{\pi} \left(2t + \tau_2 - \tau_1 - 2\sqrt{t-\tau_2}\sqrt{t-\tau_1}\right) \tag{14}$$

and Eq. (12)

$$2t - \tau_2 + \tau_1 = 2\sqrt{t-\tau_2}\sqrt{t-\tau_1}. \tag{15}$$

Equations (14) and (15), after simple algebra involving eliminating irrationalities, lead to the following equation:

$$\frac{\pi}{2} \cdot \frac{L^2}{a(\beta\Delta T)^2} = \tau_2 - \tau_1, \tag{16}$$

which is important in the sense that the time intervals in it belong only to the measurement interval (8). Expressing the quantity $\beta\Delta T$ in terms of the limiting relative change in the length of the specimen $\Delta l_0/l$ in terms of the equation

$$\beta\Delta T = \frac{\Delta l_0}{l}, \tag{17}$$

it is easy to obtain the final theoretical relation

$$a = \frac{\pi}{2} \cdot \frac{L^2 l^2}{\Delta l_0^2(\tau_2 - \tau_1)}, \tag{18}$$

which is free from the above-mentioned drawbacks. The choice of the portion of the kinetic curve $\Delta l(t)$ that represents the interval (8), is fairly simple to make, if we use a number of specific features characteristic of this part, which Eq. (16) enables us to reveal.

Indeed, the constancy of the left side of Eq. (16) in effect represents that a third time interval τ_3 exists, directly following the time τ_2 (i.e., next to it) which simultaneously obeys an equation similar to Eq. (16), namely,

$$\frac{\pi L^2}{2a(\beta\Delta T)^2} = \tau_3 - \tau_2. \tag{19}$$

From Eqs. (16) and (19) we obtain a rule which must be satisfied in the measurement interval (8) by any three adjacent time intervals, during which the specimen changes in dimensions by the same amount (more accurately, three equal time intervals following one another), namely,

$$\tau_2 = \frac{1}{2}(\tau_1 + \tau_3). \tag{20}$$

It is worth emphasizing the practical value of the rule (20) since it enables us to determine the interval (8) without carrying out any additional experimental measurements, such as measuring the temperature of the ends of the specimen (since a concrete calculation of t_1 and t_2 is not possible from relations (2) and (4) without data on the thermal conductivity, which in turn must be determined).

The optical-interference dilatometer is of undoubted experimental convenience both for determining the interval (8) and for calculating the thermal diffusivity from Eq. (18). It can be used to measure small displacements (the distance between alternate interference maxima and minima corresponds to a specimen length of a quarter of a wavelength of the light).

We determined the thermal diffusivity of copper (and confirmed the correctness of relation (20)) using a dilatometer with an optical-interference attachment for measuring small displacements using an OKG-11 helium – neon laser (wavelength 0.6328 μ [2]). An interferogram illustrating the kinetics of the thermal expansion of copper is shown in Fig. 1. It follows from the experimental data, in particular, that for a specimen length l = 15 cm and using melted Wood's

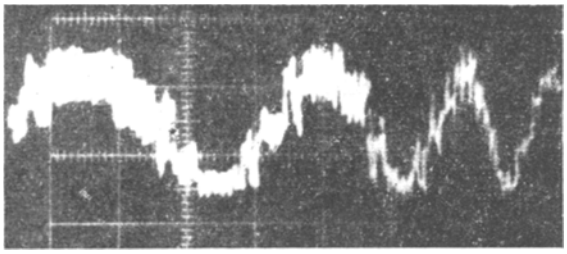

Fig. 1. Interferogram of the initial stage of the kinetics of the thermal expansion of a specimen of copper.

alloy as the heat-transmitting medium, the rule (20) begins to be satisfied 2 sec after contact is made between the Wood's alloy and one of the ends.

NOTATION

a	is the thermal diffusivity;
α	is the heat-transfer coefficient;
λ	is the thermal conductivity;
β	is the thermal-expansion coefficient;
t	is the time;
τ	is a time interval;
l	is the length of sample;
L	of the sample at some time τ;
$\Delta l(t)$	is the current value of absolute thermal change in sample dimensions in time t;
Δl_0	is the limiting absolute thermal change in dimensions;
ΔT	is the temperature jump at the heated (cooled) end;
Bi	is the Biot number.

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